

## S-band Polarimetric Radar Estimation of Snow

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**A polarimetric radar method for measuring snow has been developed. It uses a combination of radar reflectivity factor  $Z$  and specific differential phase  $K_{DP}$  to estimate snow water equivalent (SWE) rate  $S$ . The algorithm performance is demonstrated using the S-band WSR-88D radar observations for three snow cases in Virginia, Oklahoma, and Colorado. A comparison with snow gauges shows that the new method outperforms a traditional technique based on the use of a sole  $Z$ . It is important that the polarimetric algorithm yields more realistic vertical profiles of snow rate. Because  $K_{DP}$  is a polarimetric variable depending on the shape and orientation of snowflakes, the accuracy of snow estimation is contingent on realistic assumptions about these microphysical characteristics of snow. This is a challenging issue discussed in the paper.**

### Поляриметрические радиолокационные измерения снега

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*В своем недавнем исследовании Буковчич и др. (2018) использовали большой массив данных по измерению снежных осадков, полученный с помощью двумерного видео-дисдрометра в центральной Оклахоме для вывода уравнений по определению интенсивности снежных осадков  $S$ . Эти уравнения содержат удельную дифференциальную фазу  $K_{DP}$  и радарную отражаемость  $Z$  и имеют вид  $S(K_{DP}, Z) = \gamma K_{DP}^\alpha Z^\beta$ . Анализ уравнений показывает, что фактор  $\gamma$  в определенной степени зависит от изменений формы и ориентации снежинок, в то время как коэффициенты  $\alpha$  и  $\beta$  практически инвариантны по отношению к этим изменениям. Поляризационные формулы для определения снежных осадков были использованы при анализе радарных данных, полученных в разных географических регионах США: Оклахоме, Вирджинии и Колорадо. Применение поляризационных методик продемонстрировало существенное улучшение оценок снежных осадков по сравнению с традиционными  $Z$  – методами, что является обнадеживающим результатом.*

### 1. Introduction

The accurate radar measurements of snow are very challenging and difficult to accomplish. There is a high degree of natural variability in particle size distributions (PSD), snowflake densities, shapes, orientations, habits, and water content, which increases the level of complexity in remote snow measurements. Historically, the equivalent reflectivity factor at horizontal polarization ( $Z_h$ , herein  $Z$ ) has been used for snow water-equivalent rates ( $S$ ) estimation, typically in the form of power-law relations where  $Z$  is proportional to  $S^2$  (e.g., Gunn and Marshall 1958; Sekhon and Srivastava 1970; Fujiyoshi et al. 1990; Matrosov 2007, Matrosov et al., 2009; Szyrmer and Zawadzki 2010; Zhang et al. 2011; Heymsfield et al. 2016; etc.). Because the  $Z$ -based relations are sensitive to the natural PSD variability and change in particle density, the difference between various  $S(Z)$  estimates is large, producing an order of magnitude spread in  $S$  for the same values of  $Z$ .

In addition to horizontal polarization, dual-polarization radars obtain the measurements from vertical polarization plane, another source of independent information about the hydrometeor characteristics. However, there has been only limited usage of this invaluable source of information since the emergence of the dual-polarization capabilities, mainly for IWC estimation (Vivekanandan et al., 1994; Aydin and Tang, 1995; Ryzhkov et al., 1998, 2018; Lu et al., 2015; Nguyen et al., 2017, 2019).

Recently, Bukovčić et al. (2018) derived polarimetric radar relations for snow estimation (at S band, but applicable to C and X bands) from a large set of video disdrometer (2DVD) data in dry aggregated snow from Oklahoma and Colorado. They introduced a bivariate power-law relation for snow water-equivalent rate estimation based on the specific differential phase  $K_{DP}$  and reflectivity factor  $Z$ ,  $S(K_{DP}, Z) = \gamma K_{DP}^\alpha Z^\beta$ . The novel polarimetric  $S(K_{DP}, Z)$  estimate has relatively small standard deviation with respect to 2DVD estimates, in sharp contrast to a very large one from the  $S(Z)$ . On the negative side,  $S(K_{DP}, Z)$  is very sensitive to the changes in the particle density, aspect ratio, and orientation. The focus of this study is to test the viability of the novel approach of Bukovčić et al. (2018) for polarimetric snow estimation using polarimetric radar data. The 2DVD-derived  $S(K_{DP}, Z)$  relation is applied to the WSR-88D radar data in three geographical locations, Virginia, Oklahoma, and Colorado, and the results are compared to the standard  $S(Z)$  estimates and ground (in situ) measurements.

## 2. Radar data processing

Polarimetric radar measurements contain a wealth of information regarding the precipitating environment, but not all measurements are equally useful. For example, specific differential phase,  $K_{DP}$ , is a range derivative of differential phase  $\Phi_{DP}$  and can be very noisy, especially in snow. Also, the values of  $K_{DP}$  are close to zero for the irregular or aggregated snow (at S band). The emergence of new radar data displaying/processing techniques, such as Enhanced (or more appropriate “Columnar”) Vertical Profiles (EVPs, Bukovčić et al. 2017; CVPs, Murphy 2018) or Quasi Vertical Profiles (QVPs, Ryzhkov et al. 2016, Griffin et al. 2018), can help to reduce the  $K_{DP}$  measurement/estimation errors. The QVP product is radar centric and requires  $360^\circ$  azimuthal averaging. For each radial increment (range gate) within the higher tilt volume scan (usually between  $10^\circ$  and  $20^\circ$ ), a value of the radar variable averaged over a whole  $360^\circ$  circle is projected to the radar centered vertical axis. This gives QVP for a single radar scan. Repeating this procedure for all available radar temporal scans, a QVP in a time vs. height format is obtained. Hence, the QVP methodology significantly reduces the noise and improves the accuracy of  $K_{DP}$  estimate (or any other radar polarimetric variable), decreasing the measurement error to about  $0.01 \text{ deg km}^{-1}$  (decreasing the standard deviation of measurement by a factor of  $360^{1/2} \approx 19$ ). This is more than sufficient for  $K_{DP}$  to be used in snow estimation in the proximity of a radar. The QVPs are the essential data for verification of polarimetric snow relations in this study. For detailed description about the QVPs the reader is referred to Ryzhkov et al. (2016).

The QVP technique motivates another look at the existing Plan Position Indicator – PPI methodology. If the radar data from  $M$  range gates and  $N$  radials from PPI are averaged, the similar accuracy of  $K_{DP}$  as from the QVP methodology is produced (if the  $M \times N$  product is close to 360). For the location  $\sim 70 \text{ km}$  from the radar, a  $10 \times 10 \text{ km}$  box has  $\sim 320$  data points for averaging (e.g., sample spacing resolution of  $\sim 1^\circ$  in azimuth and  $0.25 \text{ km}$  in range) which is comparable to  $\sim 360$  from QVP. If the spatial scale of the process is similar to the prescribed box size, storm’s inhomogeneity/variability will have a minor impact on the accuracy of the radar estimates. The original PPI data are obtained from volume scans updated roughly every 5-6 minutes, with  $0.25 \text{ km}$  range spacing and  $0.5^\circ$ - $1^\circ$  beam width.

### 3. Verification of the polarimetric radar relations for snow on polarimetric radar data

Three cases from different geographical locations, Virginia, Oklahoma, and Colorado are presented for validation of  $S(K_{DP}, Z)$  polarimetric radar relations. The radar measurements are obtained in dry (mostly) aggregated snow during the events with one high (~55 mm), and two medium (~15 mm and ~23 mm) total snow water equivalent (SWE) accumulations. The Oklahoma  $S_{OK}(K_{DP}, Z) = 1.48K_{DP}^{0.615}Z^{0.33}$  relation from Bukovčić et al. (2018) and QVP methodology is used for verification in first two cases (herein  $S_{OK}(K_{DP}, Z)$  is denoted as  $S(K_{DP}, Z)$ ). The Plan Position Indicator (PPI) data and the Colorado relation from Bukovčić et al. (2018),  $S_{CO}(K_{DP}, Z) = 1.88K_{DP}^{0.615}Z^{0.33}$  is used in the Colorado dataset.

#### *a. 23 January 2016 east coast blizzard, Sterling, Virginia*

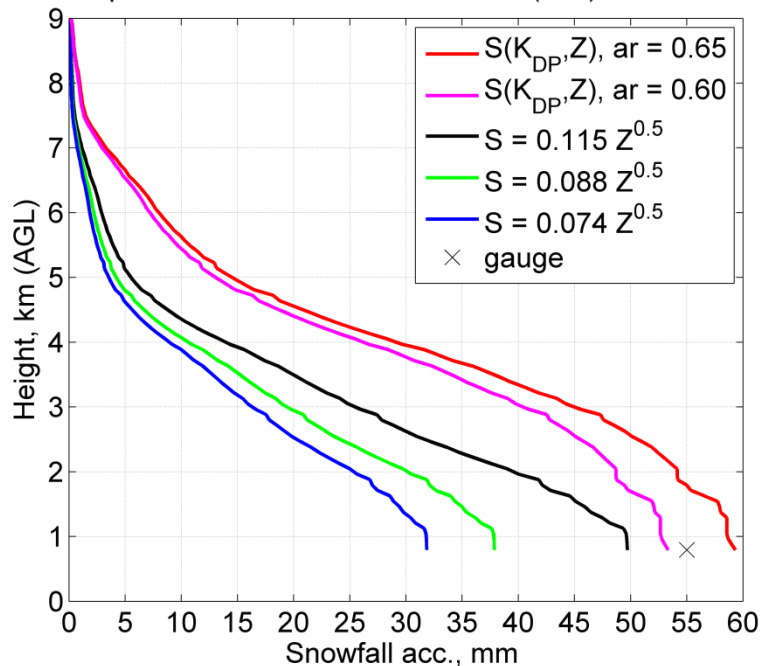
The first snowstorm used for verification, 23 January 2016 East Coast blizzard, produced about 55 mm of snow liquid-water equivalent in 24 hours. The storm hampered the day's activities and services from New York to Washington DC area, affecting an immense number of people. The maps of total snow water equivalent obtained by using the standard  $S(Z)$  relation on several WSR-88D radars (not shown) didn't match the heated gauge total accumulation. Also, some heated rain gauges showed much smaller amounts of precipitation due to partially melted or windblown snow. It is well known that widely used  $S(Z)$  relations are inaccurate because of inadequate representation of variability in snow PSDs.

Verification of the novel polarimetric snow measurement concept is presented in Fig. 1 through the comparisons of  $S(K_{DP}, Z)$  relations with collocated reference ground measurements and several standard  $S(Z)$  WSR-88D relations. The vertical profiles of total snow accumulations (Fig. 1) are obtained from KLWX QVPs (19.5° elevation angle) via multiplying  $S(Z)$  and  $S(K_{DP}, Z)$  snow rates by the time interval between the radar scans, and at the constant heights, summing the corresponding results throughout the duration of the storm. Both  $S(K_{DP}, Z)$  relations used for comparison provide better estimates of total SWE than the three  $S(Z)$  relations. The two  $S(K_{DP}, Z)$  relations are derived for different aspect ratios (the ratio of minor and major particle axis,  $ar$ ) – the red line corresponds to aspect ratio 0.65 (the Oklahoma  $S(K_{DP}, Z)$  relation), whereas the magenta line is derived for  $ar = 0.6$ . The range for the aspect ratios in aggregated snow is typically from 0.5-0.7 (Korolev and Isaac, 2003).

The X represents reference ground measurements of snow liquid-water equivalent presented at the lowest snowfall accumulation height for convenience.

Another notable feature in Fig. 1 is the “nonphysical” slope of the total SWE below the DGL (located at heights between -10°C and -20°C, at about 3-4 km AGL in Fig. 1) estimated from  $S(Z)$  relations. If saturation with respect to ice occurs below the DGL all the way to the ground, then conservation of mass is preserved (in case of no advection below the DGL). As aggregation strengthens –  $Z$  increases, and as a consequence of aggregation, the number of smaller anisotropic particles is deflated in the process –  $K_{DP}$  decreases. Thus, it is expected that total SWE estimated from  $S(Z)$  has an almost constant profile from the DGL all the way to the ground because 80% - 90% of snow is produced in the DGL. In this case,  $S(Z)$ s produce ~16, 19, and 25 mm at about 3 km AGL, which is ~50% of their total estimation at the ground level. On the other hand, both  $S(K_{DP}, Z)$  relations produce ~75% - 76% at ~3km AGL of their total amount at the ground level. Also,  $S(K_{DP}, Z)$  relations' estimates of total SWE (Fig. 1: magenta and red line) are within  $\pm 4\%$  - 7% of reference ground measurement (55 mm), whereas  $S(Z)$ s underestimate total SWE by 42%, 31%, and 10% (Fig. 1: blue, green, and black lines, respectively). Clearly,  $S(K_{DP}, Z)$  relations give physically more realistic profiles and more accurate total SWE amounts than the standard WSR-88D  $S(Z)$  relations in this case.

Vertical profiles of snowfall accumulation (mm) from 19.5° QVP

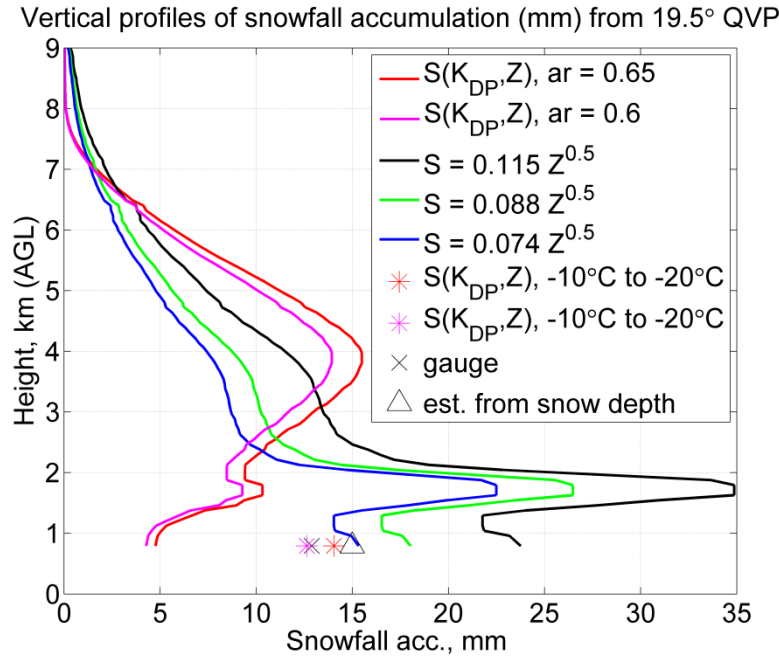


**Figure 1: Vertical profiles of total snow accumulation obtained from KLWX 19.5° QVPs using various  $S(Z)$ s and  $S(K_{DP}, Z)$  relations (red: aspect ratio –  $ar = 0.65$ , magenta:  $ar = 0.6$ ), 23 January 2016.**

*b. 1 February 2011 case, Norman, Oklahoma*

The 1 February 2011 snowstorm had a big impact on a social life and it was highly disruptive. High snow accumulations on the ground (~30-50 cm, measured by the ruler) almost completely shut down northwestern parts of the state. Central Oklahoma saw 4-8 inches (about 10-20 cm) of snow depths on the ground. The measurements of total SWE in Norman were between 12 mm and 18 mm (determined from the storm snow depth reports and converted by the 10:1 rule), about 15.3 mm on average, which is adopted as one of the ground reference measurements. The Norman Oklahoma Mesonet measurement of total SWE was ~12.9 mm.

Comparisons between the three standard  $S(Z)$  and two  $S(K_{DP}, Z)$  estimates of total SWE, obtained from KOUN QVPs, along with the ground reference measurements are shown in Fig. 2. The  $S(K_{DP}, Z)$  estimates have primary maximums in DGL (at ~4 km AGL) as opposed to  $S(Z)$  relations, with melting layer maxima at ~1.8 km AGL. This is important because 80% to 90% of snow precipitation is formed in the DGL. The hypothesis that the  $S(K_{DP}, Z)$  from DGL can be used for estimation of total SWE amount on the ground seems very plausible. Although the total SWE profile amounts estimated from  $S(K_{DP}, Z)$  are underestimated close to the ground (~5 mm), their estimates from the DGL (12.6 to 14.2 mm) are in excellent agreement with the reference ground measurements (~13 to 15 mm). The  $S(Z)$  relations display very unrealistic total SWE profiles due to contamination from the melting layer (centered at ~1.8 km AGL). However, some of the  $S(Z)$ s have total SWE estimates (~15.5 and 18 mm) at the lowest altitudes similar to the ground measurements, which in this case is rather fortuitous.



**Figure 2: Vertical profiles of total snow liquid-water equivalent accumulation obtained from KOUN 19.5° QVPs using various  $S(Z)$ s and  $S(K_{DP}, Z)$  relations (red: aspect ratio –  $ar = 0.65$ , magenta:  $ar = 0.6$ ), 1 February 2011.**

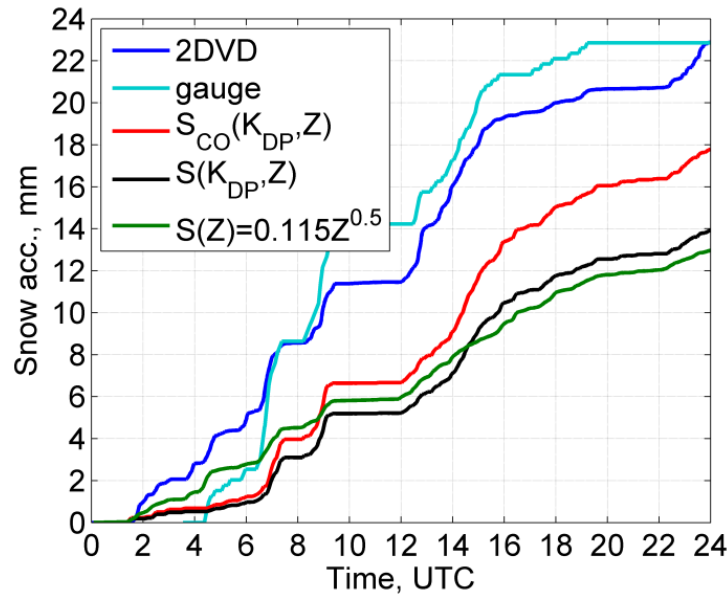
The X represents reference ground measurements of snow liquid-water equivalent from Oklahoma Mesonet, whereas  $\Delta$  is the estimate from the average snow depth measured by ruler across Norman, OK, using the 10:1 conversion rule, presented at the lowest snowfall accumulation height for convenience. Red and magenta asterisks are  $S(K_{DP}, Z)$  estimates using aspect ratios of 0.65 and 0.6 respectively, but from the DGL (-10°C to -20°C).

*c. 28 January 2013 case, Grand Mesa, Colorado*

The winter precipitation measurement experiment, funded by Water Conservation Board of Colorado, was conducted in the vicinity of Grand Mesa, CO, from January until April 2013. The reduction of the beam blockage effects from the 35°-40° azimuthal sector east of the KGJX WSR-88D radar, located in Grand Junction CO, was one of the primary goals of this experiment. This was the reason for the ground instrumentation placement in the middle of the beam blockage sector, about 21 km east from the KGJX radar. Because the blockage affected the lowest radar elevations (0.5°, 0.9°), the next available (not affected) elevation (1.29°, ~450m AGL, 3500m MSL at the instrumentation location) is used for the verification of the  $S(K_{DP}, Z)$  relations. Due to the localized nature of the storm, the data are computed as median values of 5 range gates by 3° azimuth sector (median of 30 data points) extracted directly above the reference ground measurement location. Hence, the decrease in standard deviation of polarimetric variables estimates is  $30^{1/2} \approx 5.5$  times. The case presented had the largest snow accumulation (22.9 mm SWE) during the experiment period.

Snow water equivalent accumulations from the heated gauge, 2DVD, Colorado  $S_{CO}(K_{DP}, Z) = 1.88 K_{DP}^{0.615} Z^{0.33}$ , Oklahoma  $S(K_{DP}, Z)$ , and standard  $S(Z)$  relations are presented in Fig. 3. Without taking into account the lagged gauge measurements, the  $S_{CO}(K_{DP}, Z)$  relation produced the closest SWE amount (~18 mm) to the reference measurements (~22.9 mm), about 21% smaller. Also, the Oklahoma  $S(K_{DP}, Z)$  had closer values (~14 mm) than  $S(Z)$  relation (~13 mm), although ~39% and 43% smaller in comparison to the ground reference. The estimates from the  $S(K_{DP}, Z)$ s are in accord with the difference in the relations' multipliers, which is 21% higher for the Colorado relation. The shapes of both  $S(K_{DP}, Z)$  curves resemble more the heated gauge, and especially 2DVD accumulations, than the  $S(Z)$

counterpart. This is another example of the potentially universal character of the  $S(K_{DP}, Z)$  relations, where the application to the radar data above the gauge location produced credible results.



**Figure 3: SWE accumulations from heated rain gauge (cyan line), 2DVD (blue line),  $S_{CO}(K_{DP}, Z)$  (red line),  $S(K_{DP}, Z)$  (black line), and  $S(Z)$  (green line) relations estimated from KGJX PPIs; 28 January 2013, Grand Mesa, CO.**

#### 4. Discussion

Analysis of S-band  $K_{DP}$  measurements in heavily aggregated dry snow suggests that  $K_{DP}$  is usually noisy and very low. Due to the inverse proportionality to the wavelength, these  $K_{DP}$  characteristics, prominent in aggregated snow at S-band, have smaller negative impact on the C- and especially X-band measurements. The corresponding relations for S band, and at shorter wavelengths, can be acquired by wavelength-scaling of  $K_{DP}$ . Additional tuning of C- and X-band relations may be needed with respect to the type of snow and reference ground measurements.

The quality of radar snowfall measurements can be significantly improved if new polarimetric radar processing techniques, such as Quasi-Vertical Profiles (Ryzhkov et al. 2016; Griffin et al. 2018) and Enhanced/Column Vertical Profiles (Bukovčić et al. 2017, Murphy 2018), are utilized. These techniques require substantial azimuthal/spatial averaging to reduce the statistical error of the  $K_{DP}$  estimate. Polarimetric measurements in the dendritic growth layer suggest that the magnitude of  $K_{DP}$  within this layer is substantially higher than below the DGL, where warmer temperatures are expected (e.g., Kennedy and Rutledge, 2011; Bechini et al., 2013). These options should be further explored in a future research.

#### 5. Summary

Verification of polarimetric radar  $S(K_{DP}, Z)$  relations in three geographical regions, Virginia, Oklahoma, and Colorado via reference ground measurements and comparison with the standard  $S(Z)$  relations increases confidence in the applicability of the novel concept. The use of the same  $S(K_{DP}, Z)$  relation(s) in three distinct geographical regions (Virginia, Oklahoma, and Colorado) produced encouraging results, implying potentially universal character of these relations. There is an indication that if there is no presence of advection, wind shear or turbulence, polarimetric relations produce more realistic vertical profiles of snow rate than the standard  $S(Z)$  estimates. If these processes are present at lower levels, more

accurate estimates of  $S$  from  $S(K_{DP}, Z)$  are obtained from the dendritic growth layer, where 80% to 90% of total precipitation is produced.

The estimates of  $K_{DP}$  are extremely noisy in aggregated snow and substantial spatial averaging may be required for reliable estimation of  $K_{DP}$  (Ryzhkov and Zrníc 1998). Hence, the usability of the novel polarimetric relations for snow measurements heavily depends on the  $K_{DP}$  accuracy. Extensive spatial averaging (as in QVP or CVP), and utilization of  $K_{DP}$  estimates aloft in the DGL (centered at the  $-15^{\circ}\text{C}$  isotherm where the magnitude of  $K_{DP}$  is significantly higher than in heavily aggregated snow near the surface) or just above the freezing level, could significantly reduce the measurement errors and noisiness in  $K_{DP}$ . Under the assumption that the mass flux is conserved, projection of the  $S(K_{DP}, Z)$  values from the dendritic growth layer to the ground should produce values in better agreement with ground measurements. In addition, the instantaneous snowfall rate from polarimetric relations obtained from PPI data in Colorado show better agreement with the ground measurements in comparison to the standard  $S(Z)$  relation tuned for that region. Therefore, the use of localized averaging on PPI data may produce adequate accuracy of  $K_{DP}$  (as shown in Colorado case) and increase the usability of polarimetric relations.

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